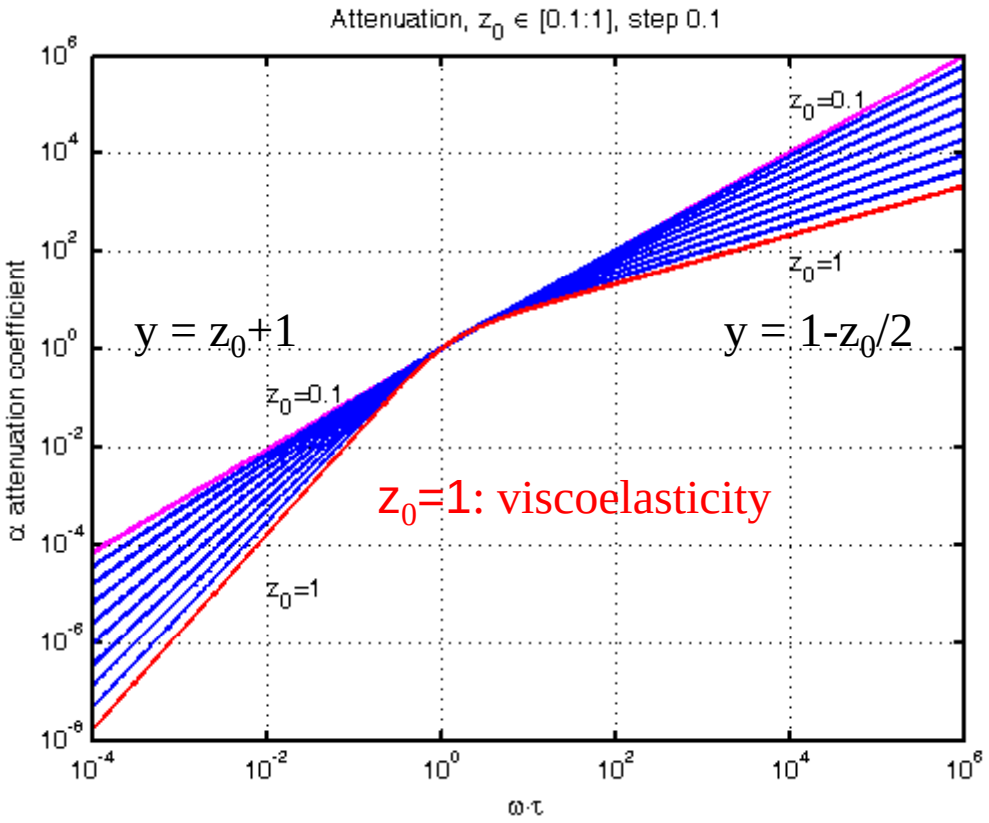
Summary Caputo equation

- Stress-strain: $T = c_{ii}S + \eta_{ii}\frac{\partial^{z_0} S}{\partial t^{z_0}}$
- Wave eq.:

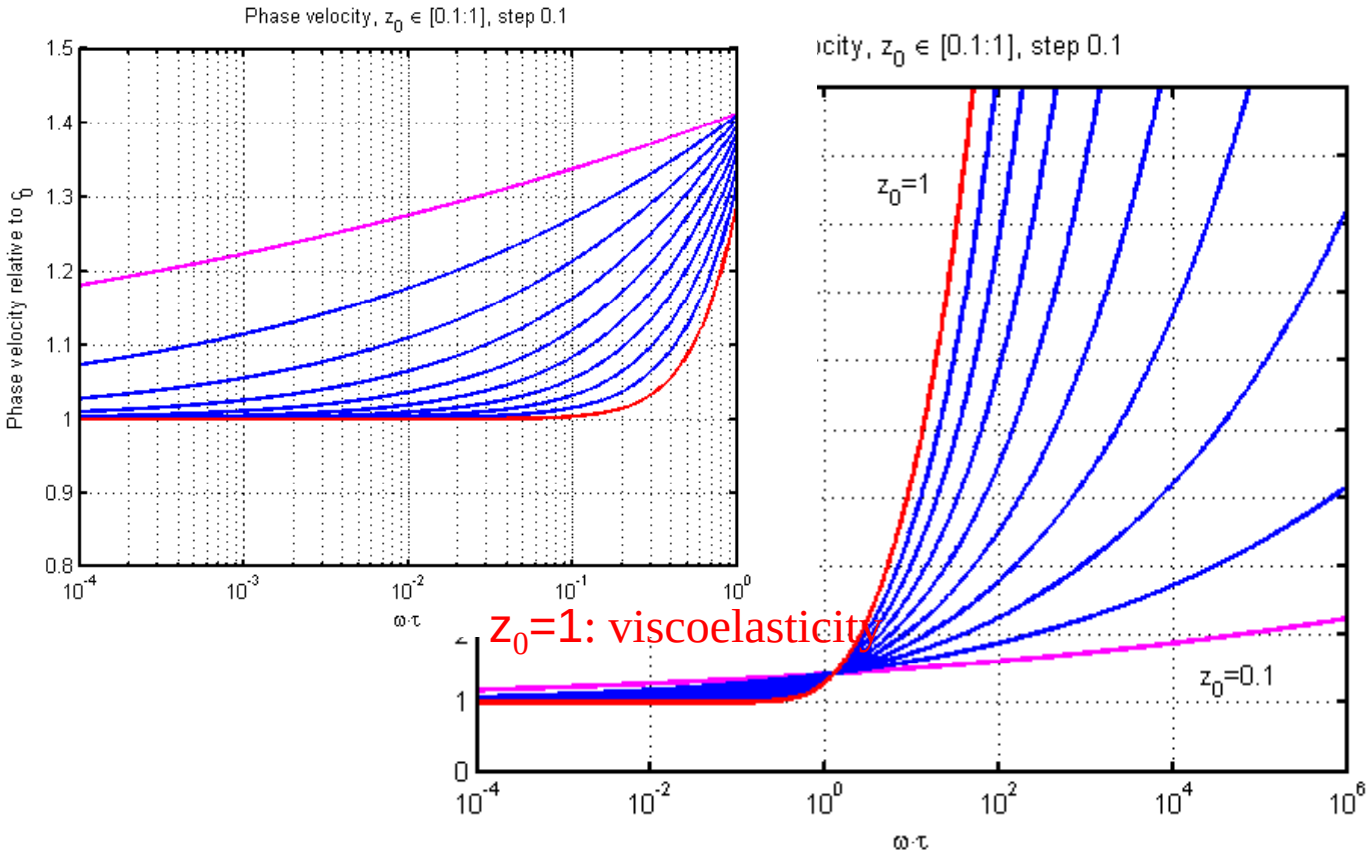
$$\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + \tau^{z_0}\frac{\partial^{z_0}}{\partial t^{z_0}}(\nabla^2 u) = 0, \tau = \frac{\eta_{ii}}{c_{ii}}, c_0^2 = c_{ii}/\rho$$

- Low-f (P-waves, low-f S): $\alpha = \alpha_0|\omega|^y, \alpha_0 = \frac{\tau^{y-1}}{2c_0}|\cos\frac{\pi y}{2}|$
  - $y = z_0+1$, $y \in (1,2]$, $z_0 \in (0,1]$
- Hi-f, S-waves: $c_0 = \sqrt{\mu/\rho}, \tau = \eta/\mu$ $\alpha = \alpha_0|\omega|^y, \alpha_0 = \frac{\tau^{y-1}}{c_0}\cos\frac{\pi y}{2}$
  - $y = 1-z_0/2$, $y \in [0,1)$, $z_0 \in (0,2]$

Attenuation



Dispersion

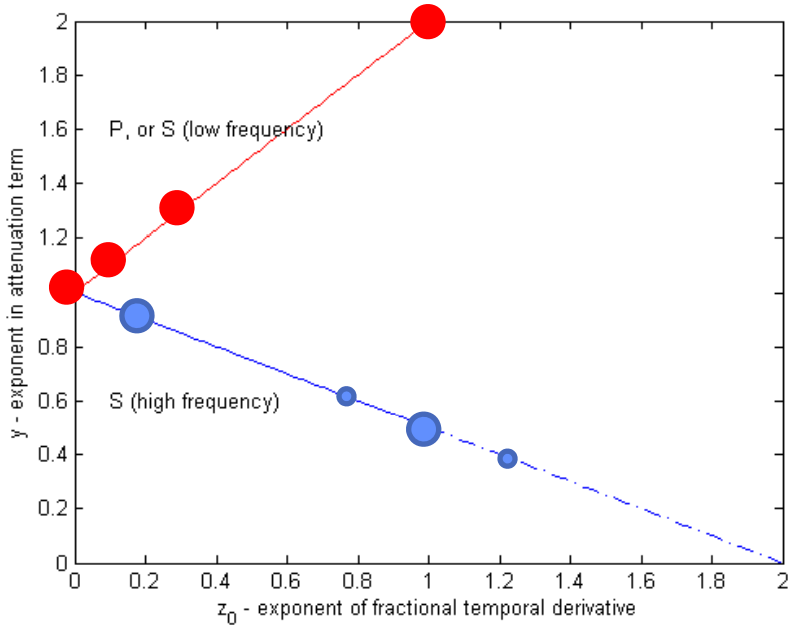


z_0=1: viscoelasticity

$y = 1-z_0/2$

$y = z_0+1$

z_0, fract. deriv. – y, exp in power law



y=2: Water,air (P), YIG (P, S)

y=1.3: Liver (P)
y=1.1: Aerogels (P)
y=1: Granite (P, S)
z_0=0.2: Living cells (S)
- •0.16-18: cortical
- •0.26-0.29: intracellular

y=0.5±0.15 : Aerogels (S)